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# The concordance correlation coefficient estimated through variance components

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**Abstract**

We demonstrate that concordance correlation coefficient (CCC) and the intraclass correlation coefficient are the same measure of agreement estimated in two ways: variance components and moment method procedures. The variance components approach allows the CCC to be easily extended to more than two observers and to be adjusted using confounding covariates by incorporating them in the mixed model

## 1. Introduction

Agreement between continuous data measured from different observers or measurement methods is a question that has received a great deal of consideration from the scientific community. The intraclass correlation coefficient (Pearson, 1901) and the concordance correlation coefficient (Lin, 1989) are two of the most popular aggregate procedures used to measure agreement when data are on a continuous scale.

The intraclass correlation coefficient (ICC) measures the amount of overall data variance due to between-subjects variability, while the concordance correlation coefficient (CCC) it is based on the distance in the plane of each pair of data to the 45º line through the origin.

Since the ICC is defined using variance components, several expressions of ICC can be found in the literature (Bartko, 1966; Shrout and Fleiss, 1979) depending on the measurement model selected to fit the data. But at the same time, this flexibility of the ICC causes confusion or misunderstanding, because the underlying measurement model is sometimes neglected. Some authors criticized the ICC as a measure of agreement among observers because it can not measure lack of accuracy (i.e.difference of means) between observers measures (Lin, 1989; Barnhart and Williamson, 2001). We will argue that the ICC is a valid measure of agreement among observers and it can really take into account the difference of observer means if the suitable expression of ICC is used.

As a result of the comparison between the CCC and ICC we will show that the former can be easily estimated by variance components. Moreover, a CCC for more than two observers and adjusted by confounding covariates would be desirable for many real problems (Lin, 1989; King and Chinchilli, 2001; Barnhart and Williamson, 2001; Barnhart, Haber and Song, 2002). From the previous result, we will show that both objectives can be achieved using the variance component estimation of CCC.

# 2. Comparison of CCC and ICC

Lin (1989) defined the CCC for two observers assuming data was distributed under a bivariant normal distribution, therefore $(Y_1, Y_2) \sim MVN(\boldsymbol{\mu}, \Sigma)$ where $Y_1$ and $Y_2$ are the array of measurements of each observer, $\boldsymbol{\mu} = (\mu_1, \mu_2)$ is the vector of the observer means and $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$ is the covariance matrix. Then the CCC expression is:

$$\rho_C = 1 - \frac{E\{(Y_1 - Y_2)^2\}}{E\{(Y_1 - Y_2)^2\} \text{when } Y_1 \text{ and } Y_2 \text{ are uncorrelated}} = \frac{2 \cdot \sigma_{12}}{\sigma_1^2 + \sigma_2^2 + (\mu_1 - \mu_2)^2}$$

Now suppose a continuous variable is measured from n subjects by k observers or judges. The measurement model assumed is $Y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$ (Fleiss, 1986), where $Y_{ij}$ is the measurement made on individual *i* by observer *j*, $\mu$ is the overall mean, $\alpha_i$ is the individual effect, $\beta_j$ is the observer effect and $e_{ij}$ is the random error. It is assumed that $\alpha_i \sim N(0, \sigma_\alpha)$ and $e_{ij} \sim N(0, \sigma_e)$ and the error term does not covary with any other component of the measurement model. The general expression of ICC is $\rho_{ICC} = \frac{\sigma_\alpha^2}{\sigma_Y^2}$ (Fleiss, 1986), where $\sigma_Y^2$ is the variance of $Y_{ij}$.

Depending on the nature of the observer effect we will choose between two expressions of $\rho_{ICC} = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_e^2}$ if the observers are considered random and their effects distributed under a normal distribution $\beta_j \sim N(0, \sigma_\beta)$ or $\rho_{ICC,2} = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_e^2}$ if observers are a fixed effect. It is obvious that $\rho_{ICC}$ takes into account the differences in average among observers whereas $\rho_{ICC,2}$ fails to do so.

At first we will consider observer effect as random if a random sample of observers has been collected, then $\rho_{ICC,2}$ should be selected, but in that case $\rho_{ICC,2}$ is not measuring agreement rather than consistency between observers (Shrout and Fleiss, 1979). Therefore, to measure agreement among observers $\rho_{ICC}$ has to be used even if the observer effect is fixed. In this case the term $\sigma_\beta^2$ will be a sum of squares $\sigma_\beta^2 = (k-1)^{-1}\sum_{j=1}^{k}\beta_j^2$ rather than a variance (Fleiss, 1986), where $\beta_j = \mu_j - \mu$ is the difference between the mean of observer *j* with respect to the overall mean and *k* is the number of observers.

To compare both coefficients we will assume that a continuous characteristic has been measured by k observers on n subjects. Assuming that observers are a fixed effect, the following equalities fulfil $\sigma_\alpha^2 = \frac{2}{k\cdot(k-1)}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\sigma_{ij}$, $\sigma_\beta^2 = \frac{1}{k\cdot(k-1)}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}(\mu_i - \mu_j)^2$ and $\sigma_e^2 = \frac{1}{k}\sum_{i=1}^{k}\sigma_i^2 - \frac{2}{k\cdot(k-1)}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\sigma_{ij}$, where $\sigma_i^2$ and $\mu_i$ are the variance and mean of the measurements made by observer i, and $\sigma_{ij}$ is the covariance between the measurements from observers *i* and *j*. Thus, the ICC can be expressed in terms of the variances, covariances and means of the observers measurements

$$\rho_{ICC} = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_e^2} = \frac{2\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\sigma_{ij}}{(k-1)\sum_{i=1}^{k}\sigma_i^2 + \sum_{i=1}^{k-1}\sum_{j=i+1}^{k}(\mu_i - \mu_j)^2}$$

which is exactly the same expression as the overall concordance correlation coefficient for k observers suggested in the works of Lin (1989), King and Chinchilli (2001) and Barnhart et al. (2002). Hence, the concordance correlation coefficient is the intraclass correlation coefficient when the observers are a fixed effect.

This result implies that CCC can be estimated by variance components through a mixed effects model easily generalizable to more than two observers and the CCC adjusted by confounding covariates can be achieved through estimating the variance components from the mixed model including those covariates.

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